WAVES & OSCILLATIONS

A PART OF

B.Sc. Physics (Hons.) Old syllabus: Paper-I

B.Sc. Physics (Hons.) Semester: I (CBCS); Course: CC-II

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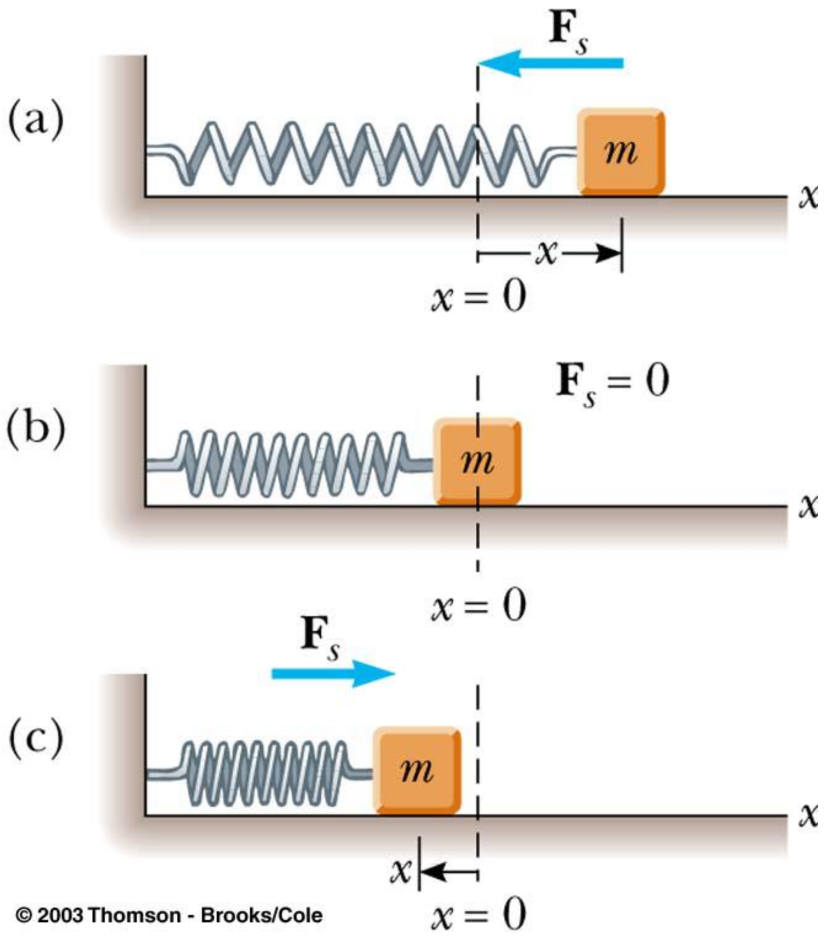
Hooke's Law Reviewed

$$F = -kx$$

- When x is positive →,
  F is negative ←;

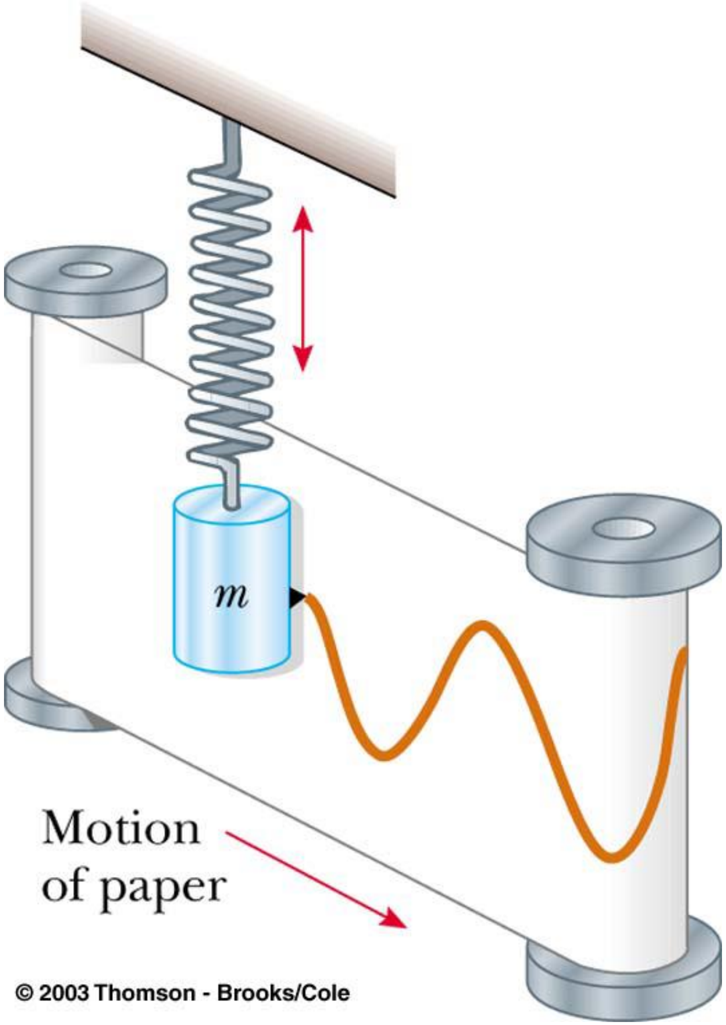
- When at equilibrium (x=0),
  F = 0 ;

- When x is negative ←,
  F is positive →;

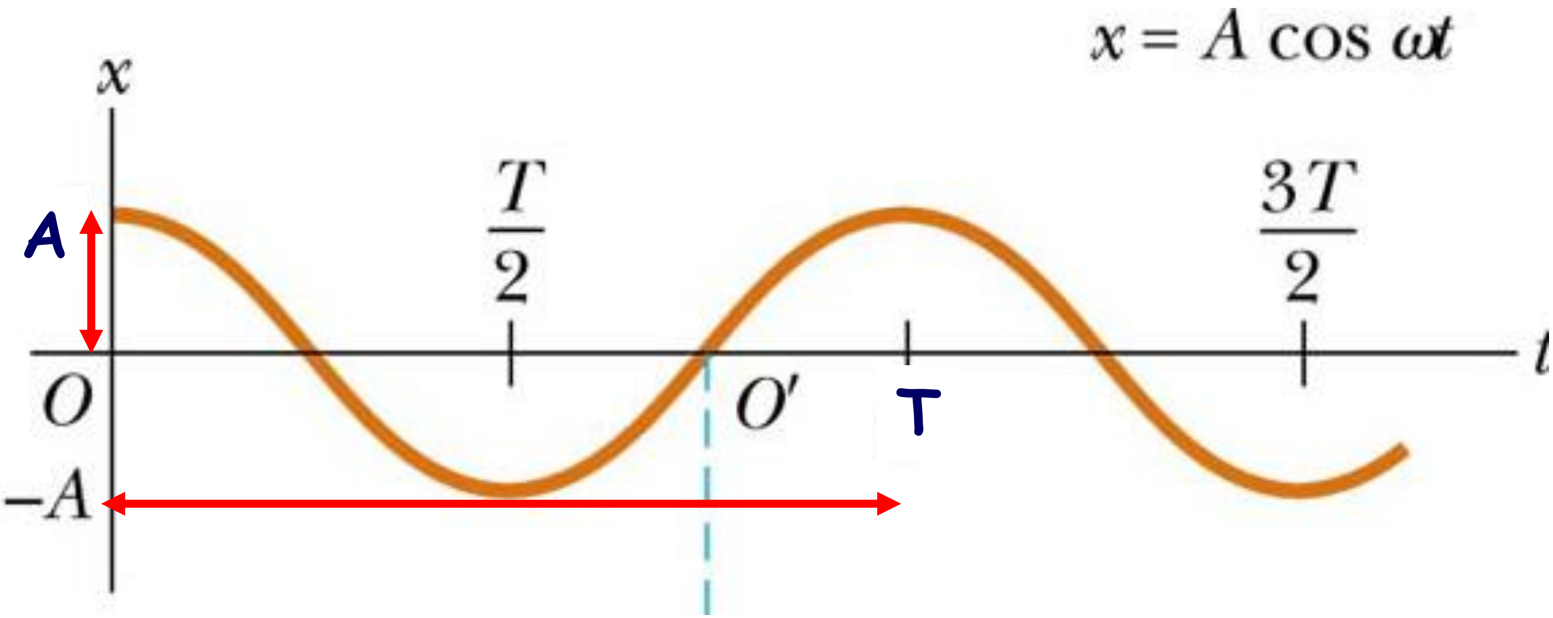


Sinusoidal Oscillation

Pen traces a sine wave



Graphing x vs. t



A : <u>amplitude</u> (length, m) T : <u>period</u> (time, s)

Some Vocabulary

$$x = A\cos(\omega t - \phi)$$
$$= A\cos(2\pi f t - \phi)$$
$$= A\cos\left(\frac{2\pi t}{T} - \phi\right)$$

$$f = \frac{1}{T}$$

$$\omega = 2\pi f = \frac{2\pi}{T}$$

f = Frequency
ω = Angular Frequency
T = Period
A = Amplitude
ϕ = phase

Phases

Phase is related to starting time

$$x = A\cos\left(\frac{2\pi t}{T} - \phi\right)$$

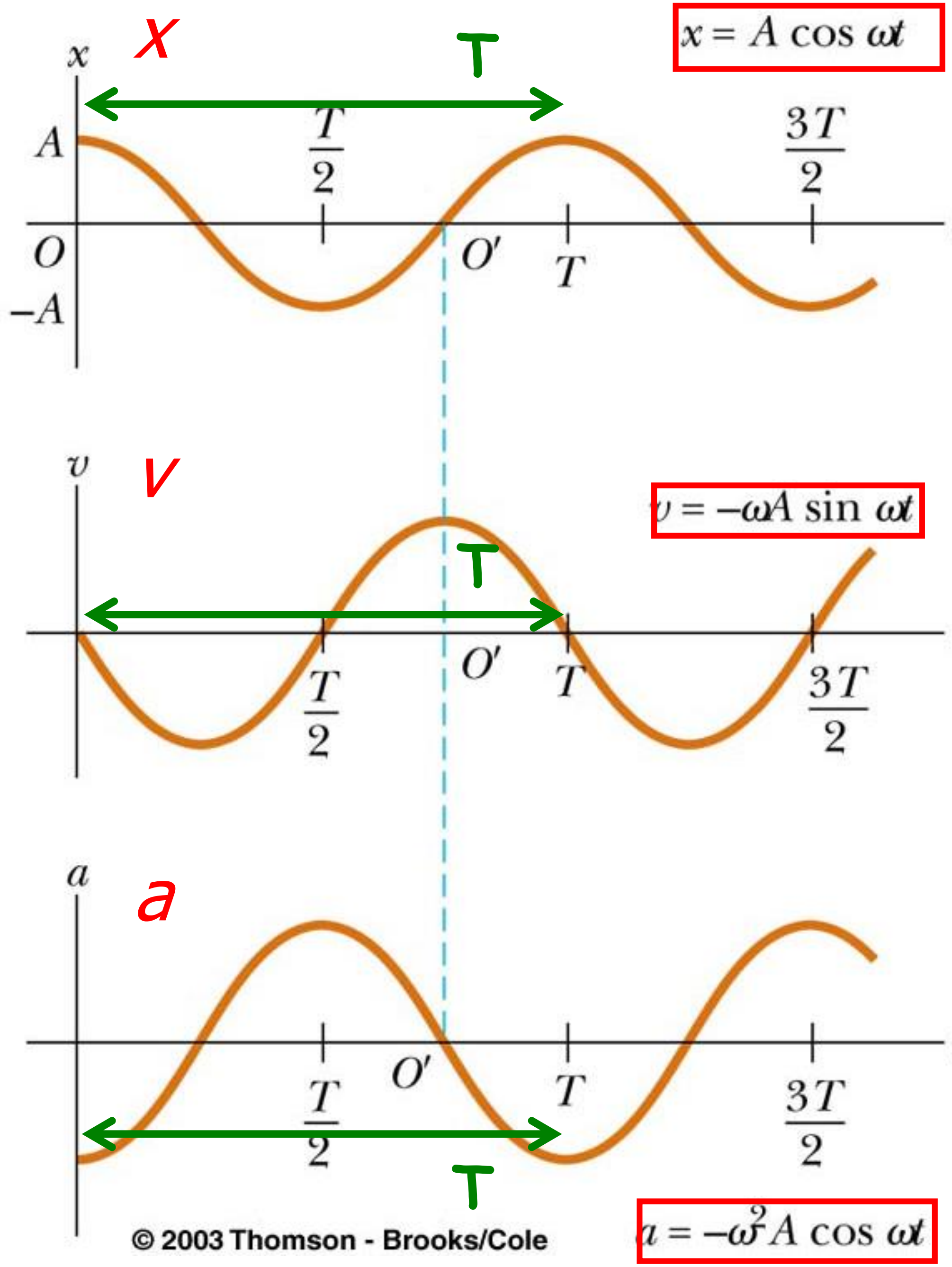
$$= A\cos\left(\frac{2\pi(t - t_0)}{T}\right) \quad \text{if } \phi = \frac{2\pi t_0}{T}$$

90-degrees changes cosine to sine

$$\cos\left(\omega t - \frac{\pi}{2}\right) = \sin(\omega t)$$

Velocity and Acceleration vs. time

- Velocity is 90° "out of phase" with x: When x is at max, v is at min ....
- Acceleration is 180° "out of phase" with x $a = F/m = -(k/m)\,x$



v and *a* vs. t

$$x = A\cos\omega t$$

$$v = -v_{max}\sin\omega t$$

$$a = -a_{max}\cos\omega t$$

Find v_{max} with E conservation

$$\frac{1}{2}kA^2 = \frac{1}{2}mv_{max}^2$$

$$v_{max} = A\sqrt{\frac{k}{m}}$$

Find a_{max} using F=ma

$$-kx = ma$$

$$-kA\cos\omega t = -ma_{max}\cos\omega t$$

$$a_{max} = A\frac{k}{m}$$

What is ω?

Requires calculus. Since

$$\frac{d}{dt} A\cos\omega t = -\omega A \sin\omega t$$

$$v_{\max} = \omega A = A\sqrt{\frac{k}{m}}$$

$$\omega = \sqrt{\frac{k}{m}}$$

Formula Summary

$$f = \frac{1}{T}$$

$$\omega = 2\pi f = \frac{2\pi}{T}$$

$$x = A\cos(\omega t - \phi)$$

$$v = -\omega A\sin(\omega t - \phi)$$

$$a = -\omega^2 A(\cos\omega t - \phi)$$

$$\omega = \sqrt{\frac{k}{m}}$$

Example 1

An block-spring system oscillates with an amplitude of 3.5 cm. If the spring constant is 250 N/m and the block has a mass of 0.50 kg, determine

(a) the mechanical energy of the system

(b) the maximum speed of the block

(c) the maximum acceleration.

a) 0.153 J

b) 0.783 m/s

c) 17.5 m/s^2

Example 2

A 36-kg block is attached to a spring of constant k=600 N/m. The block is pulled 3.5 cm away from its equilibrium positions and released from rest at t=0. At t=0.75 seconds,

a) what is the position of the block?

a) -3.489 cm

b) what is the velocity of the block?

b) -1.138 cm/s

Example 3

A 36-kg block is attached to a spring of constant k=600 N/m. The block is pulled 3.5 cm away from its equilibrium position and is pushed so that is has an initial velocity of 5.0 cm/s at t=0.

a) What is the position of the block at t=0.75 seconds?

a) -3.39 cm

Example 4a

An object undergoing simple harmonic motion follows the expression,

$$x(t) = 4 + 2\cos[\pi(t-3)]$$

Where x will be in cm if t is in seconds

The amplitude of the motion is:

a) 1 cm
b) 2 cm
c) 3 cm
d) 4 cm
e) -4 cm

Example 4b

An object undergoing simple harmonic motion follows the expression,

$$x(t) = 4 + 2\cos[\pi(t - 3)]$$

Here, x will be in cm if t is in seconds

The period of the motion is:

a) 1/3 s
b) 1/2 s
c) 1 s
d) 2 s
e) $2/\pi$ s

Example 4c

An object undergoing simple harmonic motion follows the expression,

$$x(t) = 4 + 2\cos[\pi(t-3)]$$

Here, x will be in cm if t is in seconds

The frequency of the motion is:

a) 1/3 Hz
b) 1/2 Hz
c) 1 Hz
d) 2 Hz
e) π Hz

Example 4d

An object undergoing simple harmonic motion follows the expression,

$$x(t) = 4 + 2\cos[\pi(t-3)]$$

Here, x will be in cm if t is in seconds

The angular frequency of the motion is:

a) 1/3 rad/s
b) 1/2 rad/s
c) 1 rad/s
d) 2 rad/s
e) π rad/s

Example 4e

An object undergoing simple harmonic motion follows the expression,

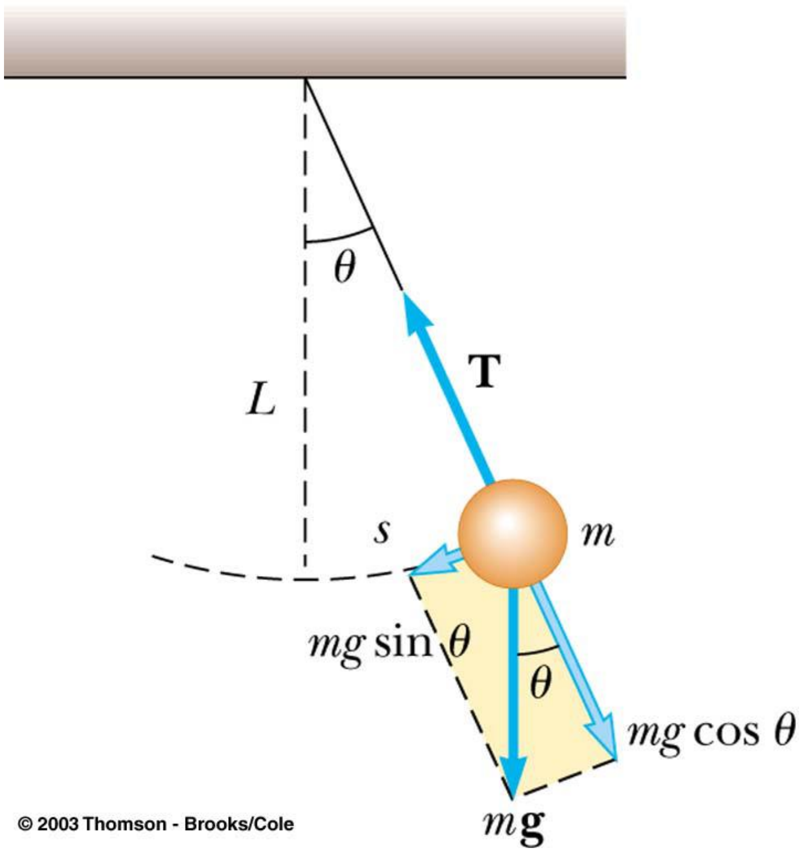
$$x(t) = 4 + 2\cos[\pi(t-3)]$$

Here, x will be in cm if t is in seconds

The object will pass through the equilibrium position at the times, t = ______ seconds

a) …, -2, -1, 0, 1, 2 …
b) …, -1.5, -0.5, 0.5, 1.5, 2.5, …
c) …, -1.5, -1, -0.5, 0, 0.5, 1.0, 1.5, …
d) …, -4, -2, 0, 2, 4, …
e) …, -2.5, -0.5, 1.5, 3.5,

Simple Pendulum



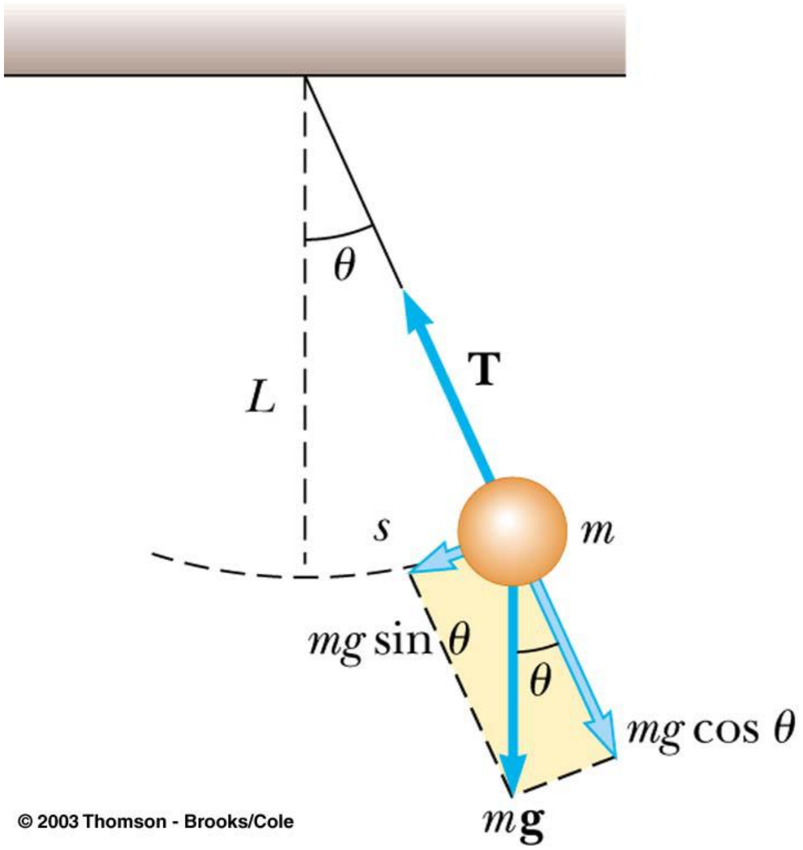
$$F = -mg\sin\theta$$

$$\sin\theta = \frac{x}{\sqrt{x^2 + L^2}} \approx \frac{x}{L}$$

$$F \approx -\frac{mg}{L}x$$

Looks like Hooke's law (k → mg/L)

Simple Pendulum



$$F = -mg\sin\theta$$

$$\sin\theta = \frac{x}{\sqrt{x^2 + L^2}} \approx \frac{x}{L}$$

$$F \approx -\frac{mg}{L}x$$

$$\omega = \sqrt{\frac{g}{L}}$$

$$\theta = \theta_{\max}\cos(\omega t - \phi)$$

Simple pendulum

$$\omega = \sqrt{\frac{g}{L}}$$

Frequency independent of mass and amplitude!
(for small amplitudes)

Example 5

A man enters a tall tower, needing to know its height h. He notes that a long pendulum extends from the roof almost to the ground and that its period is 15.5 s.

(a) How tall is the tower?

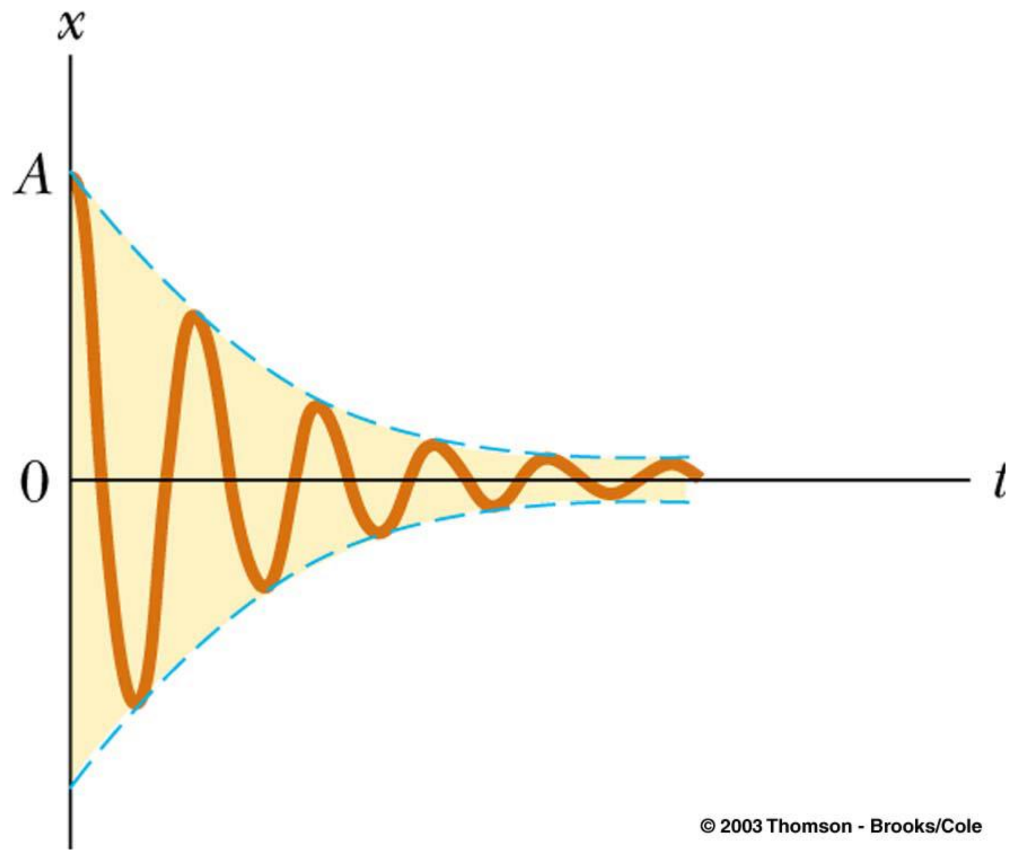
a) 59.7 m

(b) If this pendulum is taken to the Moon, where the free-fall acceleration is 1.67 m/s^2, what is the period of the pendulum there?

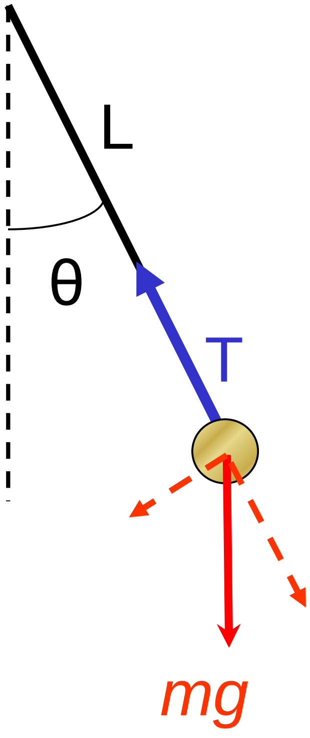
b) 37.6 s

Damped Oscillations

In real systems, friction slows motion



Simple Pendulum



Recall, for a simple pendulum we have the following equation of motion:

$$\frac{d^2\theta}{dt^2} = -\frac{g}{L}\theta$$

Which give us: $\omega = \sqrt{\frac{g}{L}}$

Hence: $$L = \frac{g}{\omega^2} = \frac{gT^2}{4\pi^2}$$

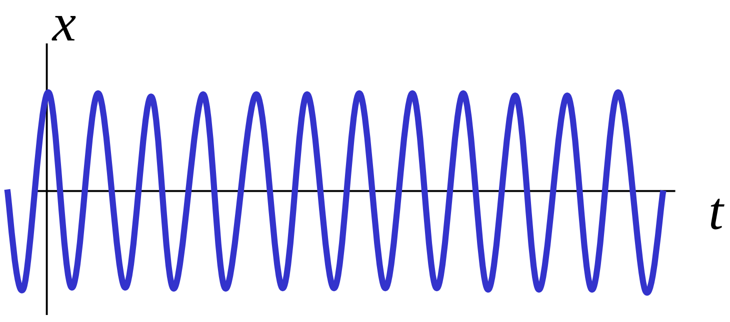
or: $$g = \omega^2 L = \frac{4\pi^2 L}{T^2}$$

Application - measuring height

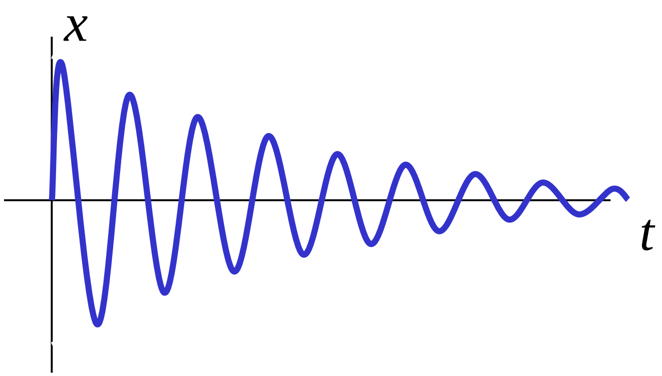
- finding variations in *g* → underground resources

SHM and DAMPED OSCILLATION

SHM: $x(t) = A \cos \omega t$
Motion continues indefinitely.
Only conservative forces act,
so the mechanical energy is
constant.



Damped oscillator: dissipative
forces (friction, air resistance, *etc.*)
remove energy from the oscillator,
and the amplitude decreases with
time.

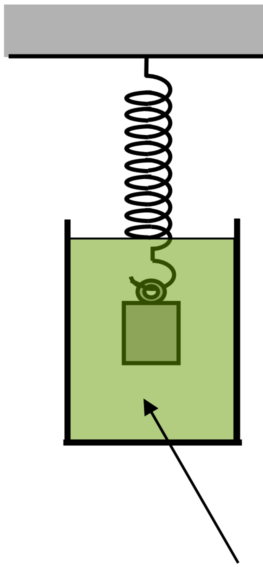


A damped oscillator has external nonconservative force(s) acting on the system. A common example is a force that is proportional to the velocity.

$\mathbf{f} = -b\mathbf{v}$ where b is a constant damping coefficient

F=ma give:

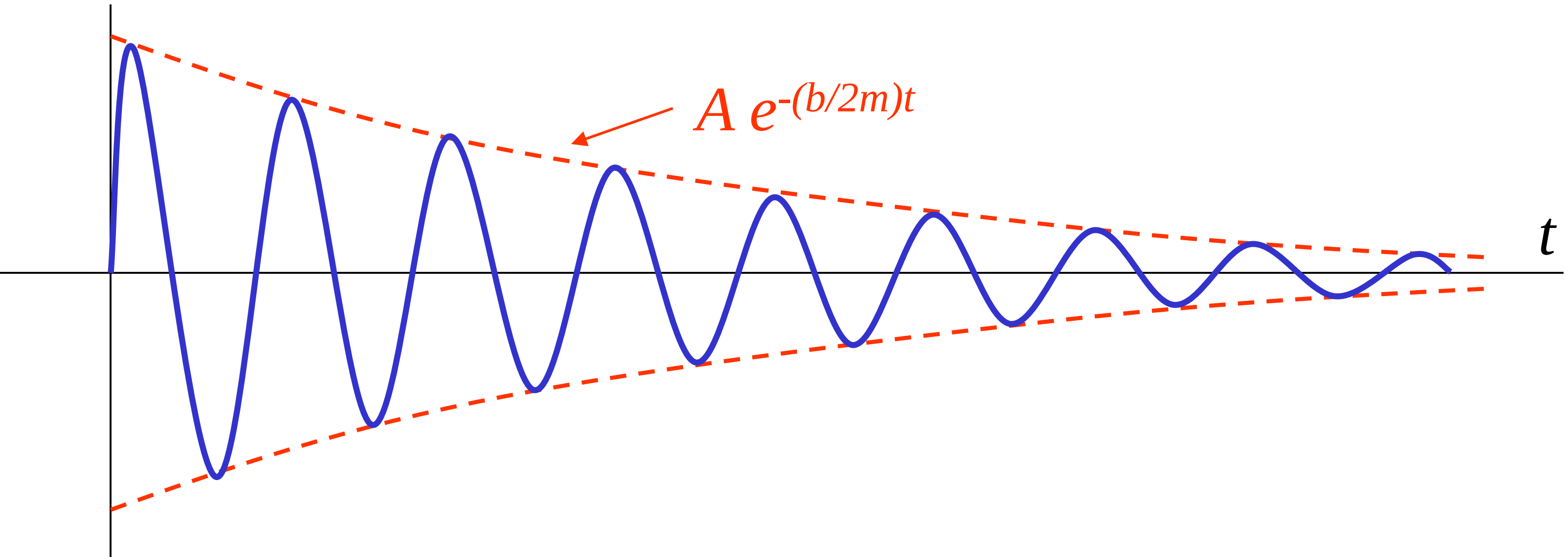
$$-kx - b\frac{dx}{dt} = m\frac{d^2x}{dt^2}$$



eg: green water (weak damping)

For weak damping (small *b*), the solution is:

$$x(t) = Ae^{-\frac{b}{2m}t}\cos(\omega t + \phi)$$



Without damping: the angular frequency is $\omega_0 = \sqrt{\dfrac{k}{m}}$

With damping: $\omega = \sqrt{\dfrac{k}{m} - \left(\dfrac{b}{2m}\right)^2} = \sqrt{\omega_0{}^2 - \left(\dfrac{b}{2m}\right)^2}$

Effectively, the frequency ω is slower with damping, and

the amplitude gets smaller (decays exponentially) as time goes on:

$$A(t) = A_o e^{-\frac{b}{2m}t}$$

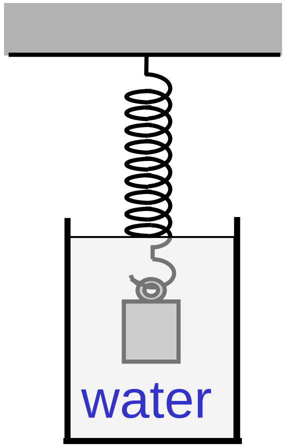
Example: A mass on a spring oscillates with initial amplitude 10 cm. After 10 seconds, the amplitude is 5 cm.

Question: What is the value of $b/(2m)$?

Question: What is the amplitude after 30 seconds?

Example: A pendulum of length 1.0 m has an initial amplitude 10°, but after a time of1000 s it is reduced to 5°. What is b/(2m)?

Types of Damping

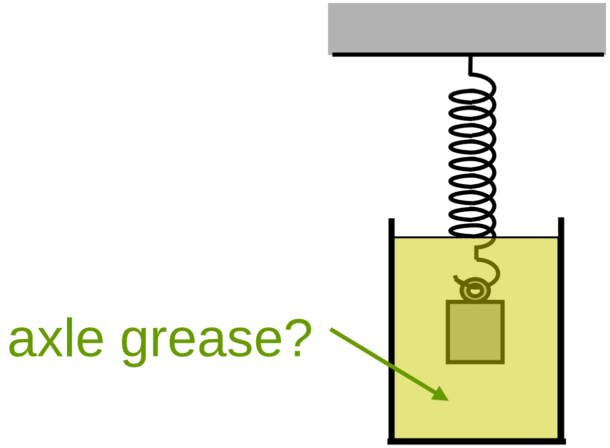


Since: $$\omega = \sqrt{\omega_0^2 - \left(\frac{b}{2m}\right)^2}$$

We can have different cases for the value under the root: >0, 0 or <0. This leads to three types of damping !

Eg: Strong damping (b large): there is **no oscillation** when:

$$\frac{b}{2m} \geq \omega_0$$

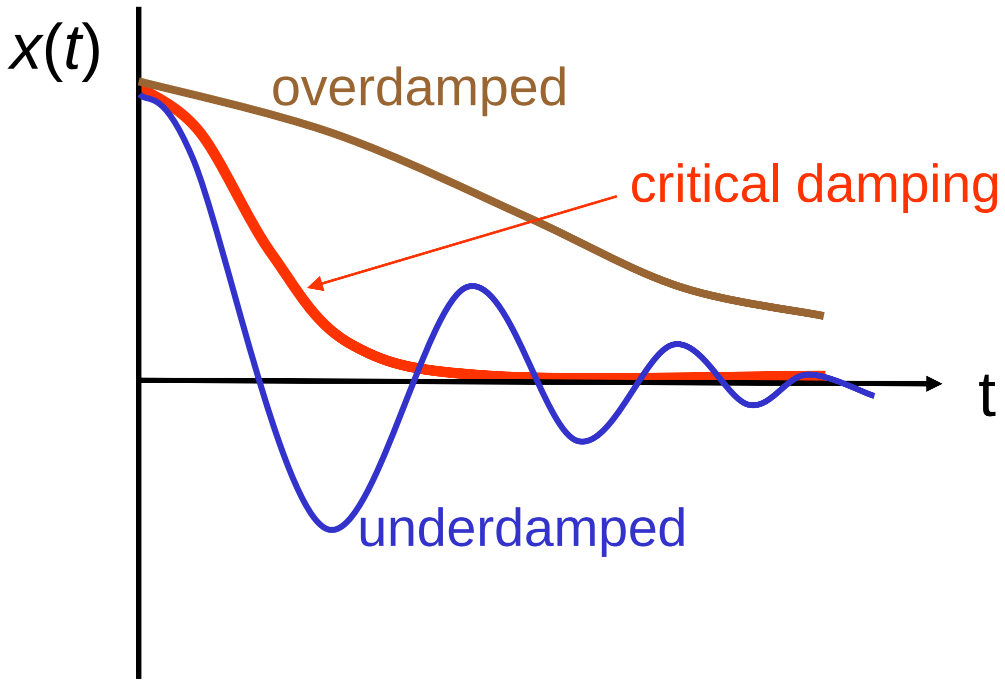


$b < 2m\omega_0$: "Underdamped", oscillations with decreasing amplitude

$b = 2m\omega_0$: "Critically damped"

$b > 2m\omega_0$: "Overdamped", no oscillation

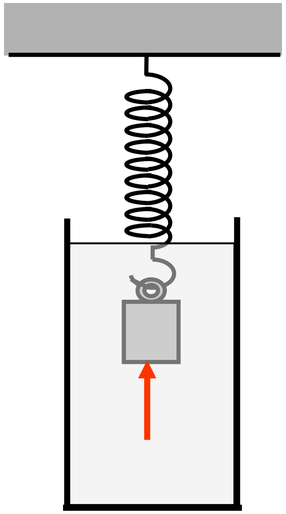
Critical damping provides the *fastest dissipation of energy*.



FORCED OSCILLATION AND RESONANCE

- Forced Oscillations
- Resonance

Forced Oscillations



A periodic, *external* force pushes on the mass (in addition to the spring and damping):

$$F_{ext}(t) = F_{\max} \cos \omega t$$

This transfers energy *into* the system

*The frequency ω is set by the machine applying the force. The system responds by oscillating at the **same frequency** ω. The amplitude can be very large if the external driving frequency is close to the "natural" frequency of the oscillator.*

$$\omega_0 = \sqrt{\frac{k}{m}}$$

is called the natural frequency or resonant frequency of the oscillation.

Newton's 2nd Law: $$\Sigma F = F_{\max} \cos \omega t - kx - b\frac{dx}{dt} = m\frac{d^2x}{dt^2}$$

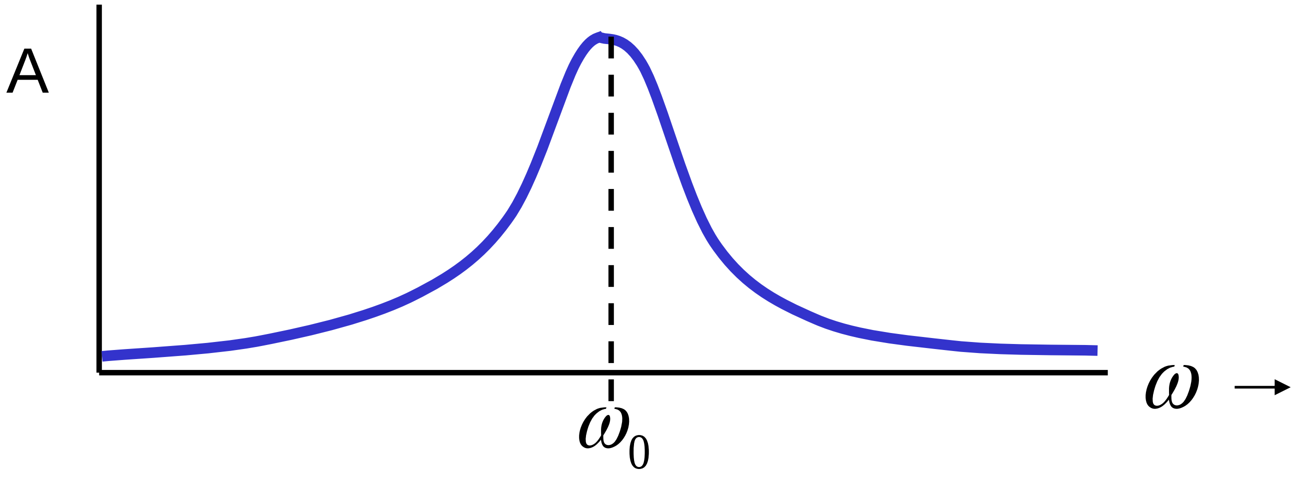
Assume that : $x = A\cos(\omega t + \phi)$; then the amplitude of a drive oscillator is given by:

$$A = \frac{F_{\max}/m}{\sqrt{\left(\omega^2 - {\omega_0}^2\right)^2 + \left(\frac{\omega b}{m}\right)^2}}$$

$$A = \frac{F_{\max}/m}{\sqrt{\left(\omega^2 - \omega_0{}^2\right)^2 + \left(\frac{\omega b}{m}\right)^2}}$$

So, as $\omega \to \omega_0$ the amplitude, *A*, increases !

If the external "push" has the same frequency as the resonant frequency ω_0. The driving force is said to be in resonance with the system.



Resonance occurs because the driving force changes direction at just the same rate as the “natural” oscillation would reverse direction, so the driving force reinforces the natural oscillation on every cycle.

Quiz: Where in the cycle should the driving force be at its maximum value for maximum average power?

A) When the mass is at maximum x (displacement)

B) When the mass is at the midpoint (x = 0)

C) It matters not

Example

A 2.0 kg object attached to a spring moves without friction and is driven by an external force given by:

$F=(3.0N)\sin(8\pi t)$

If the force constant of the spring is 20.0N/m, determine

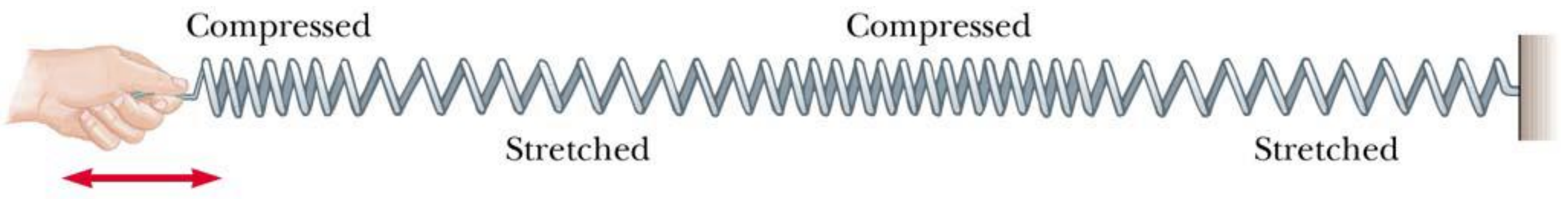
a) the period of the motion

b) the amplitude of the motion

TRAVELING WAVES

- Sound
- Surface of a liquid
- Vibration of strings
- Electromagnetic
    - Radio waves
    - Microwaves
    - Infrared
    - Visible
    - Ultraviolet
    - X-rays
    - Gamma-rays
- Gravity

Longitudinal (Compression) Waves



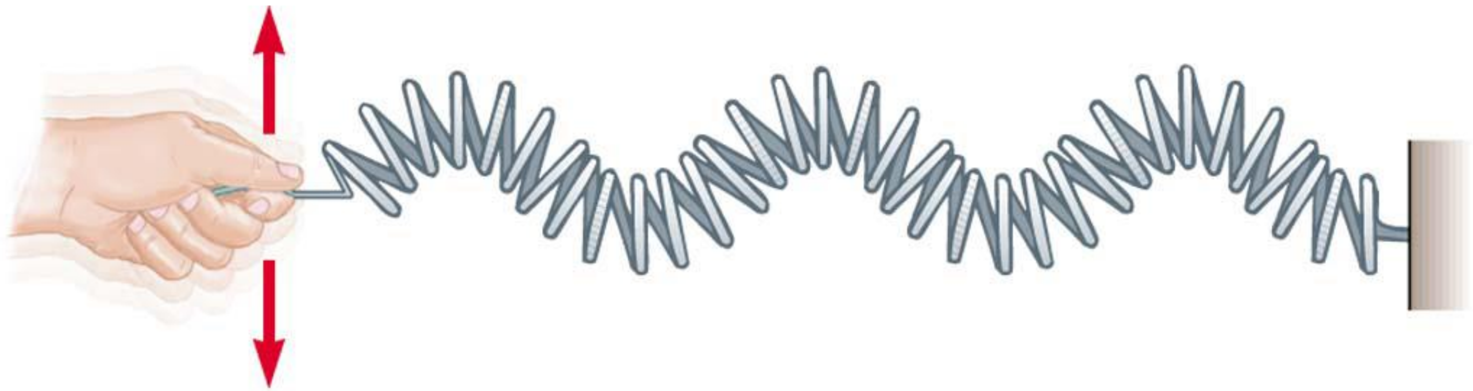
(b) Longitudinal wave

Sound waves are longitudinal waves

Transverse Waves

Elements move perpendicular to wave motion

Elements move parallel to wave motion

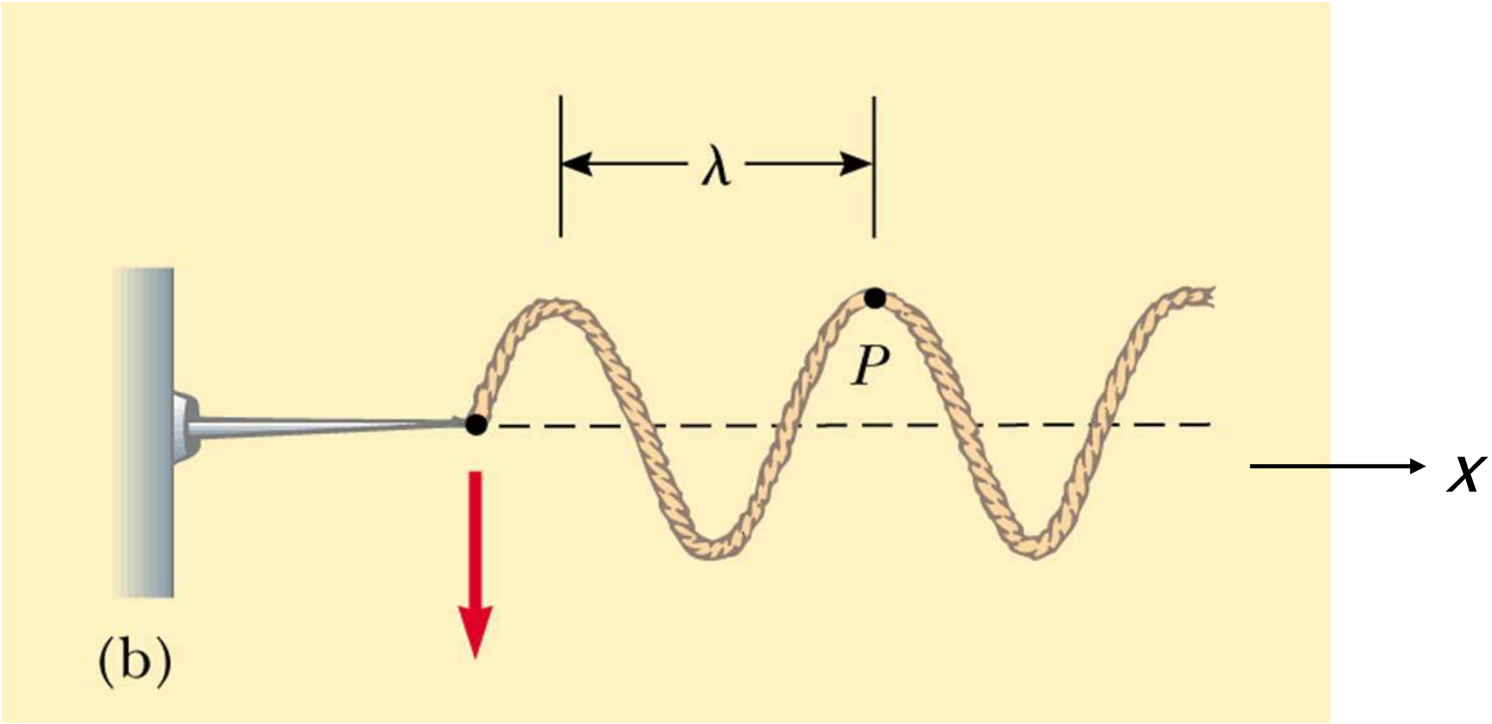


(a) Transverse wave

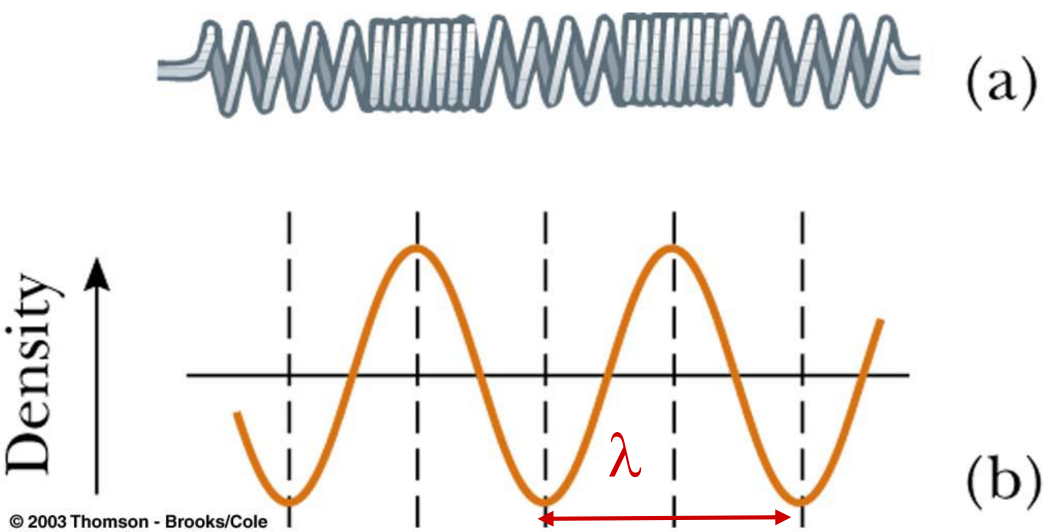
Snapshot of a Transverse Wave

$$y = A\cos\left(2\pi\frac{x}{\lambda} - \phi\right)$$

wavelength



Snapshot of Longitudinal Wave



$$y = A\cos\left(2\pi\frac{x}{\lambda} - \phi\right)$$

y could refer to pressure or density

Moving Wave

$$y = A\cos\left(2\pi\frac{x - vt}{\lambda} - \phi\right)$$

Replace x with x-vt
if wave moves to the right.
Replace with x+vt if
wave should move to left.

moves to right with velocity v

Fixing x=0,

$$y = A\cos\left(-2\pi\frac{v}{\lambda}t - \phi\right)$$

$$f = \frac{v}{\lambda}, \quad v = f\lambda$$

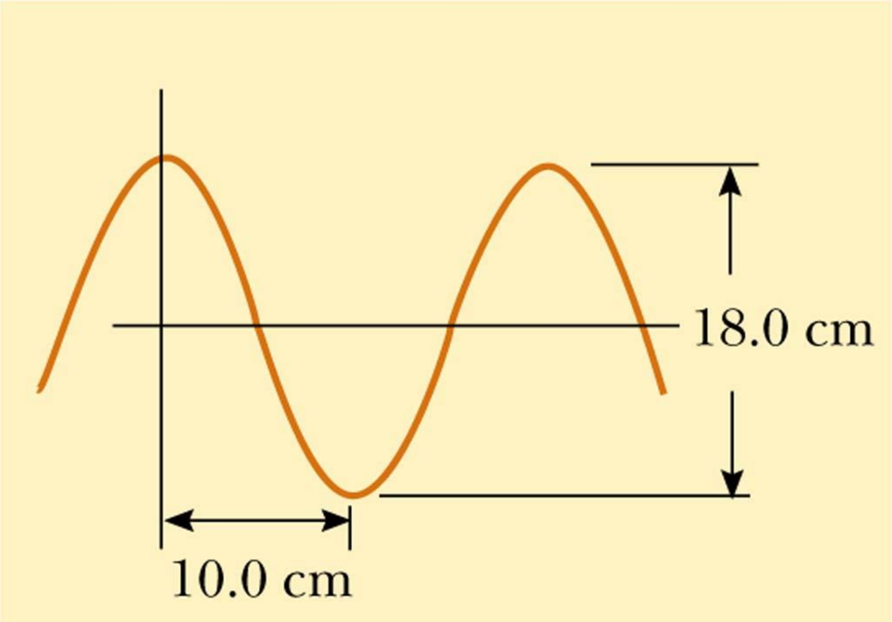
Moving Wave: Formula Summary

$$y = A\cos\left[2\pi\left(\frac{x}{\lambda}\,\mathrm{m}\,ft\right) - \phi\right]$$

$$v = f\lambda$$

Example 6a

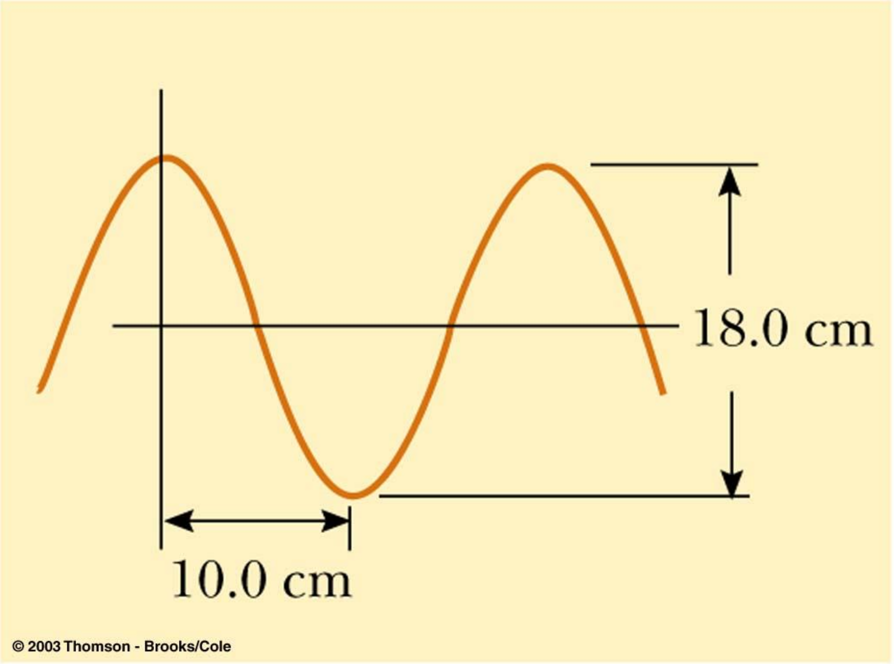
A wave traveling in the positive *x* direction has a frequency of f = 25.0 Hz as shown in the figure. The wavelength is:



a) 5 cm
b) 9 cm
c) 10 cm
d) 18 cm
e) 20 cm

Example 6b

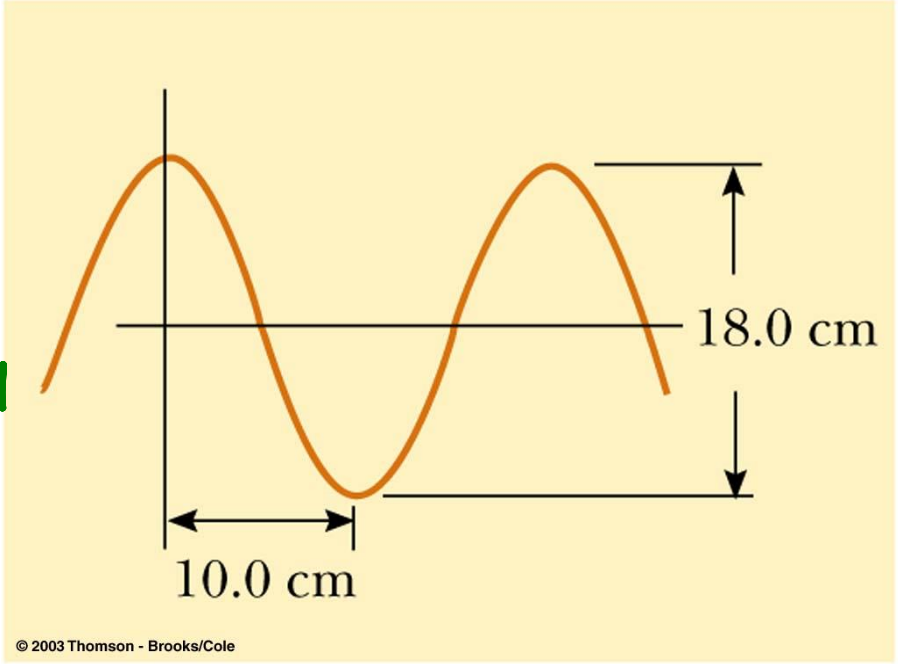
A wave traveling in the positive *x* direction has a frequency of f = 25.0 Hz as shown in the figure. The amplitude is:



a) 5 cm
b) 9 cm
c) 10 cm
d) 18 cm
e) 20 cm

Example 6c

A wave traveling in the positive *x* direction has a frequency of f = 25.0 Hz as shown in the figure. The speed of the wave is:



a) 25 cm/s
b) 50 cm/s
c) 100 cm/s
d) 250 cm/s
e) 500 cm/s

Example 7a

Consider the following expression for a pressure wave,

$$P = 60 \cdot \cos(2x - 3t)$$

where it is assumed that x is in cm, t is in seconds and P will be given in N/m^2.

What is the amplitude?

a) 1.5 N/m^2
b) 3 N/m^2
c) 30 N/m^2
d) 60 N/m^2
e) 120 N/m^2

Example 7b

Consider the following expression for a pressure wave,

$$P = 60 \cdot \cos(2x - 3t)$$

where it is assumed that x is in cm, t is in seconds and P will be given in N/m^2.

What is the wavelength?

a) 0.5 cm
b) 1 cm
c) 1.5 cm
d) π cm
e) 2π cm

Example 7c

Consider the following expression for a pressure wave,

$$P = 60 \cdot \cos\left(2x - 3t\right)$$

where it is assumed that x is in cm, t is in seconds and P will be given in N/m^2.

What is the frequency?

a) 1.5 Hz
b) 3 Hz
c) $3/\pi$ Hz
d) $3/(2\pi)$ Hz
e) $3\pi/2$ Hz

Example 7d

Consider the following expression for a pressure wave,

$$P = 60 \cdot \cos(2x - 3t)$$

where it is assumed that x is in cm,t is in seconds and P will be given in N/m^2.

What is the speed of the wave?

a) 1.5 cm/s
b) 6 cm/s
c) 2/3 cm/s
d) $3\pi/2$ cm/s
e) $2/\pi$ cm/s

Example 8

Which of these waves move in the positive x direction?

$$1)\, y = -21.3 \cdot \cos(3.4x + 2.5t)$$

$$2)\, y = -21.3 \cdot \cos(3.4x - 2.5t)$$

$$3)\, y = -21.3 \cdot \cos(-3.4x + 2.5t)$$

$$4)\, y = -21.3 \cdot \cos(-3.4x - 2.5t)$$

$$5)\, y = 21.3 \cdot \cos(3.4x + 2.5t)$$

$$6)\, y = 21.3 \cdot \cos(3.4x - 2.5t)$$

$$7)\, y = 21.3 \cdot \cos(-3.4x + 2.5t)$$

$$8)\, y = 21.3 \cdot \cos(-3.4x - 2.5t)$$

a) 5 and 6
b) 1 and 4
c) 5,6,7 and 8
d) 1,4,5 and 8
e) 2,3,6 and 7

Speed of a Wave in a Vibrating String

$$v = \sqrt{\frac{T}{\mu}} \quad \text{where} \quad \mu = \frac{m}{L}$$

For different kinds of waves: (e.g. sound)

- Always a square root
- Numerator related to restoring force
- Denominator is some sort of mass density

Example 9

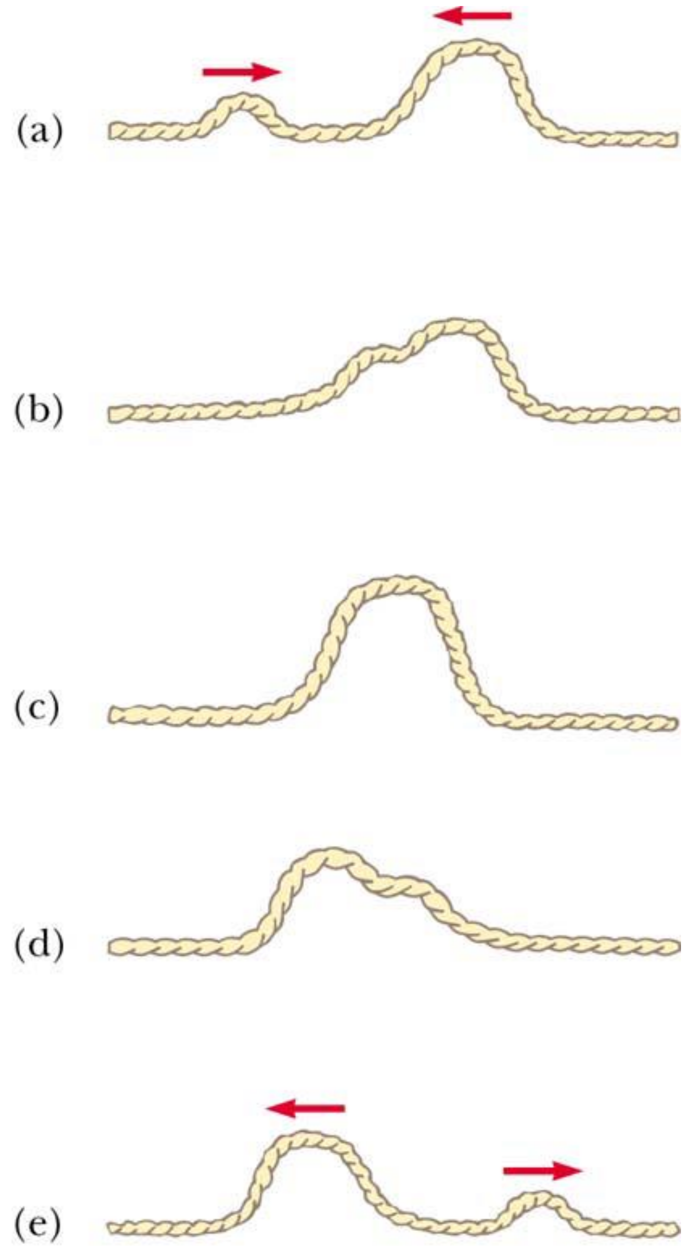
A string is tied tightly between points *A* and *B* as a communication device. If one wants to double the wave speed, one could:

a) Double the tension
b) Quadruple the tension
c) Use a string with half the mass
d) Use a string with double the mass
e) Use a string with quadruple the mass

Superposition Principle

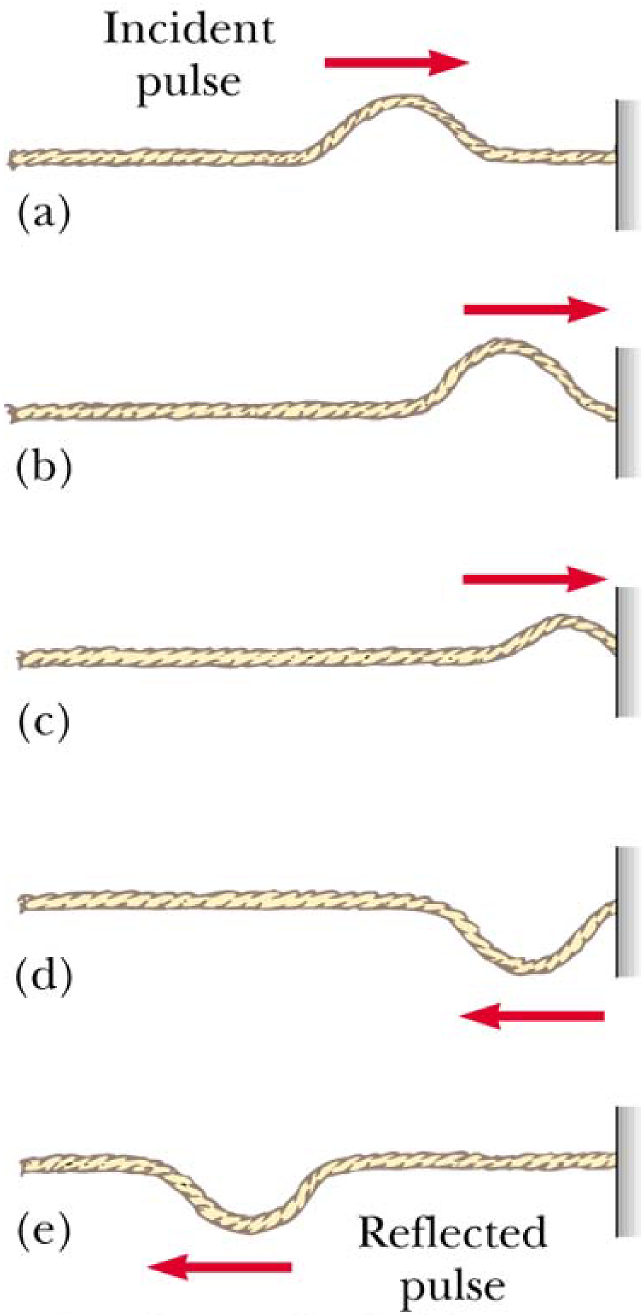
Traveling waves can pass through each other without being altered.

$$y(x,t) = y_1(x,t) + y_2(x,t)$$



Reflection – Fixed End

Reflected wave is inverted



Reflection – Free End

Reflected pulse not inverted